Mathematics for Business A

Step-by-Step Solutions

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My Story

- I am 25-year old guy from Serbia, and I went to **the best math high school in Europe.**
- I did my BSc in Mathematics, had **a 93% GPA**, and stacked up a bunch of awards.
- I graduated with an MSc in Statistics and Data Science from KU Leuven — **as the only recipient of a full scholarship** — and earned **magna cum laude** honors.
- I have done hundreds of tutoring sessions, helping dozens of BBA students succeed, **nearly all of them passed their exams.**

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Question

Let x stand for any strictly positive number. Write the following expressions as a single power of x.

(a) $\left(x^{0.5}\right)^3$

(b) $x^2\sqrt[3]{x}$

(c) $\left(\frac{1}{x^{0.25}}\right)^6$

(d) $\frac{x^3\sqrt{x}}{\sqrt[4]{x^3}}$

Solution

(a) Simplify $\left(x^{0.5}\right)^3$

Tip : Power of a Power Rule

To raise a power to another power, you multiply the exponents. The general formula is:

$$(a^m)^n = a^{m \cdot n}$$

We apply this rule to the given expression:

$$\left(x^{0.5}\right)^3 = x^{0.5 \cdot 3} = x^{1.5}$$

The exponent can also be written as a fraction: $1.5 = \frac{3}{2}$.

Answer: The expression simplifies to $x^{1.5}$ or $x^{\frac{3}{2}}$.

(b) Simplify $x^2 \sqrt[3]{x}$

Tip : Rules for Roots and Multiplication

Two rules are needed to simplify this expression:

- **Roots as Fractional Exponents:** A root can be expressed as a power with a fractional exponent.

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

- **Product Rule:** When multiplying powers with the same base, you add their exponents.

$$a^m \cdot a^n = a^{m+n}$$

First, convert the cube root to its fractional exponent form:

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

Now, apply the product rule to combine the terms:

$$x^2 \cdot x^{\frac{1}{3}} = x^{2+\frac{1}{3}} = x^{\frac{6}{3}+\frac{1}{3}} = x^{\frac{7}{3}}$$

Answer: The expression simplifies to $x^{\frac{7}{3}}$.

(c) Simplify $\left(\frac{1}{x^{0.25}}\right)^6$

Tip : Rules for Fractions and Powers

This problem uses two main rules:

- **Negative Exponent Rule:** A term in the denominator can be written in the numerator by making its exponent negative.

$$\frac{1}{a^m} = a^{-m}$$

- **Power of a Power Rule:** To raise a power to another power, multiply the exponents.

$$(a^m)^n = a^{m \cdot n}$$

First, use the negative exponent rule to rewrite the fraction:

$$\frac{1}{x^{0.25}} = x^{-0.25}$$

Next, apply the power of a power rule:

$$\left(x^{-0.25}\right)^6 = x^{-0.25\cdot 6} = x^{-1.5}$$

The exponent can also be written as a fraction: $-1.5 = -\frac{3}{2}$.

Answer: The expression simplifies to $x^{-1.5}$ or $x^{-\frac{3}{2}}$.

(d) Simplify $\frac{x^3\sqrt{x}}{\sqrt[4]{x^3}}$

Tip : Combining Exponent Rules

To solve this, we combine several exponent rules in sequence:

- **Fractional Exponents for Roots:** Convert all roots to powers using the rule $\sqrt[n]{a^m} = a^{\frac{m}{n}}$.
- **Product Rule:** Simplify the numerator by adding exponents: $a^m \cdot a^n = a^{m+n}$.
- **Quotient Rule:** Simplify the fraction by subtracting exponents: $\frac{a^m}{a^n} = a^{m-n}$.

First, convert all roots to their fractional exponent forms:

$$\sqrt{x} = x^{\frac{1}{2}} \quad \text{and} \quad \sqrt[4]{x^3} = x^{\frac{3}{4}}$$

Substitute these back into the expression:

$$\frac{x^3 \cdot x^{\frac{1}{2}}}{x^{\frac{3}{4}}}$$

Second, use the product rule to combine the terms in the numerator:

$$x^3 \cdot x^{\frac{1}{2}} = x^{3+\frac{1}{2}} = x^{\frac{6}{2}+\frac{1}{2}} = x^{\frac{7}{2}}$$

Third, use the quotient rule to combine the numerator and denominator:

$$\frac{x^{\frac{7}{2}}}{x^{\frac{3}{4}}} = x^{\frac{7}{2}-\frac{3}{4}} = x^{\frac{14}{4}-\frac{3}{4}} = x^{\frac{11}{4}}$$

Answer: The expression simplifies to $x^{\frac{11}{4}}$.

Question

Simplify the following expressions.

(a) $\left(x^2y^3\right)^4$

(b) $x^{\frac{2}{3}}y^{\frac{1}{2}}x^{\frac{3}{4}}y^{\frac{1}{3}}$

(c) $\left(\frac{x}{yz}\right)^2\left(\frac{y}{xz}\right)^2\left(\frac{z}{xy}\right)^2$

Solution

(a) Simplify $\left(x^2y^3\right)^4$

Tip : Power of a Product Rule

When a product of terms is raised to a power, the exponent is distributed to each term inside the parentheses. The general rule is:

$$(a^m b^n)^p = a^{m\cdot p}\cdot b^{n\cdot p}$$

We apply this rule by multiplying the outer exponent (4) with each of the inner exponents:

$$\left(x^2y^3\right)^4 = x^{2\cdot 4}y^{3\cdot 4} = x^8y^{12}$$

Answer: x^8y^{12}

(b) Simplify $x^{\frac{2}{3}}y^{\frac{1}{2}}x^{\frac{3}{4}}y^{\frac{1}{3}}$

Tip : Product Rule for Exponents

To multiply terms with the same base, you add their exponents. This is applied separately for each base.

$$a^m\cdot a^n = a^{m+n}$$

First, group the terms with the same base together:

$$\left(x^{\frac{2}{3}}x^{\frac{3}{4}}\right)\cdot\left(y^{\frac{1}{2}}y^{\frac{1}{3}}\right)$$

Now, add the exponents for each base. To add the fractions, we find a common denominator.

$$x^{\frac{2}{3}+\frac{3}{4}} = x^{\frac{8}{12}+\frac{9}{12}} = x^{\frac{17}{12}}$$
$$y^{\frac{1}{2}+\frac{1}{3}} = y^{\frac{3}{6}+\frac{2}{6}} = y^{\frac{5}{6}}$$

Combining these gives the final expression.

Answer: $x^{\frac{17}{12}}y^{\frac{5}{6}}$

(c) Simplify $\left(\frac{x}{yz}\right)^2 \left(\frac{y}{xz}\right)^2 \left(\frac{z}{xy}\right)^2$

Tip : Simplifying Expressions with Fractional Powers

This problem involves several rules:

- **Power of a Fraction:** Distribute the exponent to the numerator and the denominator: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.
- **Product and Quotient Rules:** Combine the resulting fractions by multiplying numerators with numerators, and denominators with denominators. Then simplify by subtracting exponents of like bases: $\frac{a^m}{a^n} = a^{m-n}$.

First, distribute the exponent of 2 into each of the three terms:

$$\left(\frac{x^2}{y^2z^2}\right) \cdot \left(\frac{y^2}{x^2z^2}\right) \cdot \left(\frac{z^2}{x^2y^2}\right)$$

Next, combine these fractions by multiplying all numerators together and all denominators together:

$$\frac{x^2y^2z^2}{y^2z^2 \cdot x^2z^2 \cdot x^2y^2} = \frac{x^2y^2z^2}{x^4y^4z^4}$$

Finally, simplify the expression by subtracting the exponents of the corresponding bases:

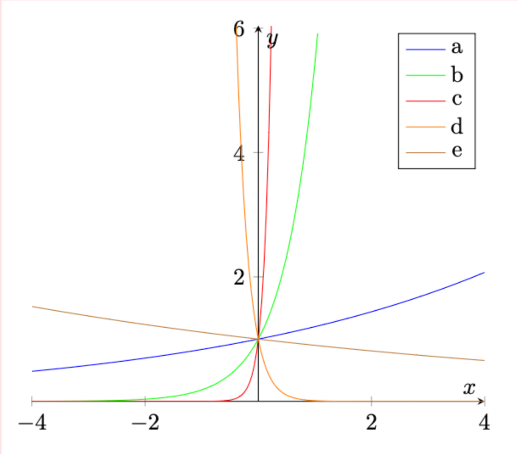
$$x^{2-4}y^{2-4}z^{2-4} = x^{-2}y^{-2}z^{-2}$$

Answer: $x^{-2}y^{-2}z^{-2}$

Chapter 1 - Question 3

Question

The graphs of five exponential functions are drawn in the following figure. The bases that were used are 2500, 0.01, 1.2, $2e$, and 0.9. Find out which graph corresponds to which exponential function.



Solution

Tip : Identifying Graphs of Exponential Functions

Exponential functions $y = a^x$ behave differently based on their base a:

- If $a > 1$, the graph increases as x increases.
- If $0 < a < 1$, the graph decreases as x increases.
- Larger bases grow faster; smaller bases (closer to 1) grow slower.
- Bases smaller than 1 also decay faster if they are close to 0.

Let's match each graph to its base based on the given figure:

1. The function c corresponds to 2500: The base is very large, so the graph grows extremely fast.
2. The function d corresponds to 0.01: The base is very small, so the graph decays extremely fast.
3. The function a corresponds to 1.2: The base is slightly larger than 1, so the graph grows slowly.
4. The function b corresponds to $2e$: This is a moderately large base, so the graph

grows faster than a.

5. The function e corresponds to 0.9: The base is slightly smaller than 1, so the graph decays slowly.

Answer:

- Function **(a)** has a base of 1.2.
- Function **(b)** has a base of $2e$.
- Function **(c)** has a base of 2500.
- Function **(d)** has a base of 0.01.
- Function **(e)** has a base of 0.9.

Chapter 1 - Question 4

Question

A function f has an equation of the form:

$$y = A \cdot b^x,$$

where A and b are real numbers. The graph of f contains the points $(0.5, 1)$ and $(2.5, 9)$.

Determine the equation of f, i.e., find the values of A and b.

Solution

Tip : Exponential Functions

For an exponential function $y = A \cdot b^x$:

- A is the initial value (the value of y when $x = 0$).
- b is the base, determining the growth or decay rate.

To find A and b, substitute the given points into the equation and solve step by step.

Step 1: Write equations for the two given points.
Substitute $(0.5, 1)$ into $y = A \cdot b^x$:

$$1 = A \cdot b^{0.5}.$$

Substitute $(2.5, 9)$ into $y = A \cdot b^x$:

$$9 = A \cdot b^{2.5}.$$

Step 2: Eliminate A by dividing the equations.
Divide the second equation by the first:

$$\frac{9}{1} = \frac{A \cdot b^{2.5}}{A \cdot b^{0.5}} \quad \Rightarrow \quad 9 = b^2.$$

Solve for b:

$$b = 3.$$

Step 3: Solve for A using $b = 3$.
Substitute $b = 3$ into the first equation:

$$1 = A \cdot 3^{0.5}.$$

Simplify:

$$A = \frac{1}{\sqrt{3}}.$$

Tip : Alternative Method to Find A

The same result for A can be obtained by substituting $b = 3$ into the second equation:

$$9 = A \cdot 3^{2.5}.$$

Simplify to solve for A:

$$A = \frac{9}{3^{2.5}} = \frac{1}{\sqrt{3}}.$$

Step 4: Write the final equation.
The equation of f is:

$$f(x) = \frac{1}{\sqrt{3}} \cdot 3^x,$$

or equivalently:

$$f(x) = 3^{x-0.5}.$$

Answer:

$$A = \frac{1}{\sqrt{3}}, \quad b = 3, \quad f(x) = \frac{1}{\sqrt{3}} \cdot 3^x \quad \text{or} \quad f(x) = 3^{x-0.5}.$$

Question

If possible, calculate the following without the use of a calculator.

(a) $\log_2 1$

(b) $\log_2 \left(\frac{1}{8}\right)$

(c) $\log_2 \left(\sqrt[4]{25}\right)$

(d) $\log_2 \left(\frac{1}{\sqrt[3]{2}}\right)$

(e) $\log_{\frac{1}{3}} 9$

(f) $\log_3 10$

(g) $\log(1\,000\,000)$

(h) $\log(10^{12})$

(i) $\log(0.001)$

(j) $\ln 1$

(k) $\ln \sqrt{e}$

(l) $\ln \left(\frac{1}{e}\right)$

Solution

(a) $\log_2 1$

Tip : Logarithm of One

The logarithm of 1 is always 0, regardless of the base (as long as the base is a positive number not equal to 1).

$$\log_a(1) = 0$$

Applying this rule directly:

$$\log_2 1 = 0$$

Answer: 0

(b) $\log_2\left(\frac{1}{8}\right)$

Tip : Evaluating Logarithms

To find the value of $\log_a(b)$, ask the question: "To what power must I raise the base a to get the number b?" If you can write b as a power of a, such as $b = a^x$, then the answer is x.

$$\log_a(a^x) = x$$

We want to evaluate $\log_2\left(\frac{1}{8}\right)$. The base is 2. We need to express $\frac{1}{8}$ as a power of 2.

$$\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$$

So, the expression becomes:

$$\log_2(2^{-3}) = -3$$

Answer: -3

(c) $\log_2\left(\sqrt[4]{25}\right)$

This expression cannot be simplified to a whole number or simple fraction without a calculator because 25 is not a power of the base 2.
Answer: Cannot be computed without a calculator.

(d) $\log_2\left(\frac{1}{\sqrt[3]{2}}\right)$

Tip : Roots and Negative Exponents

Remember that roots can be written as fractional exponents and fractions can be written with negative exponents.

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad \text{and} \quad \frac{1}{a^m} = a^{-m}$$

The base is 2. We rewrite $\frac{1}{\sqrt[3]{2}}$ as a power of 2:

$$\frac{1}{\sqrt[3]{2}} = \frac{1}{2^{1/3}} = 2^{-1/3}$$

Therefore, the logarithm is:

$$\log_2(2^{-1/3}) = -\frac{1}{3}$$

Answer: $-\frac{1}{3}$

(e) $\log_{\frac{1}{3}} 9$

We need to express the number 9 as a power of the base $\frac{1}{3}$.

$$9 = 3^2 = \left(\frac{1}{3}\right)^{-2}$$

So, the expression becomes:

$$\log_{\frac{1}{3}} \left(\left(\frac{1}{3}\right)^{-2}\right) = -2$$

Answer: -2

(f) $\log_3 10$

This cannot be simplified without a calculator because 10 is not a power of the base 3.

Answer: Cannot be computed without a calculator.

(g) $\log(1\,000\,000)$

Tip : The Common Logarithm

When no base is written for 'log', it is assumed to be the common logarithm, which has a base of 10.

$$\log(x) \equiv \log_{10}(x)$$

The rule $\log_{10}(10^x) = x$ is very useful.

We rewrite $1,000,000$ as a power of 10:

$$\log(1\,000\,000) = \log_{10}(10^6) = 6$$

Answer: 6

(h) $\log(10^{12})$

Using the common logarithm rule $\log(10^x) = x$:

$$\log(10^{12}) = 12$$

Answer: 12

(i) $\log(0.001)$

We rewrite 0.001 as a power of 10:

$$\log(0.001) = \log(10^{-3}) = -3$$

Answer: -3

(j) $\ln 1$

Tip : The Natural Logarithm

The natural logarithm, written as 'ln', has a base of $e \approx 2.718$.

$$\ln(x) \equiv \log_e(x)$$

Key properties are $\ln(1) = 0$, $\ln(e) = 1$, and $\ln(e^x) = x$.

Using the property $\ln(1) = 0$:

$$\ln 1 = 0$$

Answer: 0

(k) $\ln \sqrt{e}$

First, rewrite the square root as a power of e:

$$\sqrt{e} = e^{\frac{1}{2}}$$

Now, apply the natural log:

$$\ln(e^{\frac{1}{2}}) = \frac{1}{2}$$

Answer: 0.5

(l) $\ln\left(\frac{1}{e}\right)$

First, rewrite the fraction as a power of e:

$$\frac{1}{e} = e^{-1}$$

Now, apply the natural log:

$$\ln(e^{-1}) = -1$$

Answer: -1

Question

Find the value of x:

(a) $\log_2 x = 4$

(b) $\log_x 49 = 2$

(c) $\ln(x+1) = 7$

Solution

(a) Solve $\log_2 x = 4$

Tip : Solving for the Argument

To solve a logarithmic equation of the form $\log_a(x) = b$, you can convert it to its equivalent exponential form:

$$x = a^b$$

This isolates the variable x.

The given equation is $\log_2 x = 4$. Here, the base $a = 2$ and the value $b = 4$. Converting to exponential form gives:

$$x = 2^4$$

Calculating the result:

$$x = 16$$

Answer: $x = 16$

(b) Solve $\log_x 49 = 2$

Tip : Solving for the Base

To solve for the base in an equation like $\log_x(b) = c$, convert it to its exponential form:

$$b = x^c$$

Remember that the base of a logarithm must be positive and not equal to 1 ($x > 0, x \neq 1$).

The equation is $\log_x 49 = 2$. Converting to exponential form gives:

$$49 = x^2$$

To solve for x, we take the square root of both sides:

$$x = \pm\sqrt{49} = \pm 7$$

Since the base x must be positive, we take the positive solution.
Answer: $x = 7$

(c) Solve $\ln(x + 1) = 7$

Tip : Solving Natural Logarithm Equations

The natural logarithm ln has a base of e. An equation of the form $\ln(A) = b$ can be solved by converting it to its exponential form:

$$A = e^b$$

The equation is $\ln(x+1) = 7$. The argument is $A = x+1$. Converting to exponential form:

$$x + 1 = e^7$$

Now, we solve for x by subtracting 1 from both sides:

$$x = e^7 - 1$$

This is the exact answer. A numerical approximation is often helpful.
Answer: $x = e^7 - 1 \approx 1095.63$

Chapter 1 - Question 7

Question

Calculate (using the calculator):

(a) $\log(25)$

(b) $\log(40)$

(c) $\log(625)$

(d) Now explain:

i. why the sum of the logarithms in part (a) and (b) is equal to 3,

ii. why the logarithm in part (c) is twice the logarithm in part (a).

Solution

(a) Calculate log(25)

Using a calculator, we find the value of $\log(25)$.

Answer: $\log(25) \approx 1.39794$

(b) Calculate log(40)

Similarly, we use a calculator to find the value of $\log(40)$.

Answer: $\log(40) \approx 1.60206$

(c) Calculate log(625)

Using a calculator, we find the value of $\log(625)$.

Answer: $\log(625) \approx 2.79588$

(d.i) Explain why $\log(25) + \log(40) = 3$

Tip : Logarithm Product Rule

The logarithm of a product is the sum of the logarithms of its factors. The formula is:

$$\log(a) + \log(b) = \log(a \cdot b)$$

We apply the product rule to the left side of the equation:

$$\log(25) + \log(40) = \log(25 \cdot 40)$$

Calculating the product inside the logarithm:

$$25 \cdot 40 = 1000$$

So the expression becomes:

$$\log(1000) = \log(10^3) = 3$$

Answer: This is true because $\log(25) + \log(40) = \log(25 \cdot 40) = \log(1000) = 3$.

(d.ii) Explain why $\log(625) = 2 \cdot \log(25)$

Tip : Logarithm Power Rule

The logarithm of a number raised to a power is the exponent times the logarithm of the number.

$$\log(a^n) = n \cdot \log(a)$$

We can express 625 as a power of 25:

$$625 = 25^2$$

Now, we take the logarithm of both sides and apply the power rule:

$$\log(625) = \log(25^2) = 2 \cdot \log(25)$$

Using the values from parts (a) and (c) confirms this:

$$2.79588 \approx 2 \cdot 1.39794$$

Answer: This is true because $625 = 25^2$, so $\log(625) = \log(25^2) = 2 \cdot \log(25)$.

Chapter 1 - Question 8

Question

Write as a single logarithm:

(a) $\ln 3 + \ln 7 - \ln 2 - 2 \cdot \ln 4$

(b) $0.5 \cdot (\log 225 + 8 \cdot \log 6 - 3 \cdot \log 169)$

Solution

(a) Simplify $\ln 3 + \ln 7 - \ln 2 - 2 \cdot \ln 4$

Tip : Properties for Combining Logarithms

To combine multiple logarithmic terms into a single logarithm, we use the following rules:

- **Product Rule:** $\ln(a) + \ln(b) = \ln(a \cdot b)$
- **Quotient Rule:** $\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$
- **Power Rule:** $n \cdot \ln(a) = \ln(a^n)$

First, we apply the power rule to the last term:

$$2 \cdot \ln 4 = \ln(4^2) = \ln(16)$$

Now the expression is:

$$\ln 3 + \ln 7 - \ln 2 - \ln 16$$

We can combine this into a single logarithm. Terms being added go in the numerator, and terms being subtracted go in the denominator.

$$\ln\left(\frac{3 \cdot 7}{2 \cdot 16}\right) = \ln\left(\frac{21}{32}\right)$$

Answer: $\ln\left(\frac{21}{32}\right)$

(b) Simplify $0.5 \cdot (\log 225 + 8 \cdot \log 6 - 3 \cdot \log 169)$

We use the same logarithmic properties as in part (a).

Step 1: Apply the power rule inside the parentheses.

$$8 \cdot \log 6 = \log(6^8) \quad \text{and} \quad 3 \cdot \log 169 = \log(169^3)$$

The expression becomes:

$$0.5 \cdot (\log 225 + \log(6^8) - \log(169^3))$$

Step 2: Combine the terms inside the parentheses into a single logarithm.

$$0.5 \cdot \log\left(\frac{225 \cdot 6^8}{169^3}\right)$$

Step 3: Apply the outer 0.5 coefficient using the power rule.

$$\log\left(\left(\frac{225 \cdot 6^8}{169^3}\right)^{0.5}\right) = \log\left(\frac{(225 \cdot 6^8)^{0.5}}{(169^3)^{0.5}}\right)$$

Step 4: Simplify the expression. The exponent 0.5 means taking the square root.

- Numerator: $(225 \cdot 6^8)^{0.5} = \sqrt{225} \cdot \sqrt{6^8} = 15 \cdot 6^4$
- Denominator: $(169^3)^{0.5} = (\sqrt{169})^3 = 13^3$

Substituting these back gives the final simplified form.

Answer: $\log\left(\frac{15 \cdot 6^4}{13^3}\right)$

Chapter 1 - Question 9

Question

Calculate without using the graphical calculator:

(a) $\log 1 + \log 1000$

(b) $\log_7(7^8)$

(c) $\log_3\left(\frac{27}{81}\right)$

(d) $\ln e + \log\left(\frac{1}{10}\right)$

(e) $\log_6 54 - \log_6 9$

Solution

(a) Simplify $\log 1 + \log 1000$

Tip : Logarithm Product Rule

The sum of two logarithms with the same base can be written as a single logarithm of their product.

$$\log_a(x) + \log_a(y) = \log_a(x \cdot y)$$

Using the product rule, we combine the two terms:

$$\log 1 + \log 1000 = \log(1 \cdot 1000) = \log(1000)$$

Since the common logarithm has a base of 10, and $1000 = 10^3$, the expression simplifies:

$$\log(10^3) = 3$$

Answer: 3

(b) Simplify $\log_7(7^8)$

Tip : Logarithm Inverse Property

A logarithm of a number with the same base is the inverse operation of exponentiation. This means the function simplifies to the exponent.

$$\log_a(a^x) = x$$

Applying this rule directly to the expression:

$$\log_7(7^8) = 8$$

Answer: 8

(c) Simplify $\log_3\left(\frac{27}{81}\right)$

A good first step is often to simplify the expression inside the logarithm.

$$\frac{27}{81} = \frac{1}{3}$$

Now the problem is to evaluate $\log_3\left(\frac{1}{3}\right)$. We can express $\frac{1}{3}$ as a power of the base 3:

$$\frac{1}{3} = 3^{-1}$$

So the expression becomes:

$$\log_3(3^{-1}) = -1$$

Answer: -1

(d) Simplify $\ln e + \log\left(\frac{1}{10}\right)$

Tip : Basic Logarithm Identities

Remember the definitions of the natural log ('ln', base 'e') and the common log ('log', base 10):

$$\ln(e) = 1 \quad \text{and} \quad \log(10) = 1$$

We evaluate each term separately.

$$\ln e = 1$$

$$\log\left(\frac{1}{10}\right) = \log(10^{-1}) = -1$$

Now we add the results:

$$1 + (-1) = 0$$

Answer: 0

(e) Simplify $\log_6(54) - \log_6(9)$

Tip : Logarithm Quotient Rule

The difference of two logarithms with the same base can be written as a single logarithm of their quotient.

$$\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$$

Using the quotient rule, we combine the terms:

$$\log_6(54) - \log_6(9) = \log_6\left(\frac{54}{9}\right)$$

Simplify the fraction inside the logarithm:

$$\log_6(6)$$

Since the base and the argument are the same, the result is 1.

Answer: 1

Chapter 1 - Question 10

Question

Compute $\log_3 27\underbrace{000\cdots000}_{\text{100 zeroes}}$, where the number in the logarithm consists of 27 followed by 100 zeroes.

Solution

The number is 27 followed by 100 zeros, which is $27 \cdot 10^{100}$. We want to find its logarithm in base 3. First, apply the logarithm product rule:

$$\log_3(27 \cdot 10^{100}) = \log_3(27) + \log_3(10^{100})$$

We can evaluate each part of the sum separately:

- The first term is $\log_3(27) = 3$, because $3^3 = 27$.
- For the second term, we use the power rule: $\log_3(10^{100}) = 100 \cdot \log_3(10)$.

Combining these gives the exact expression: $3 + 100 \cdot \log_3(10)$. To get a final number, a calculator gives $\log_3(10) \approx 2.096$.

Now:

$$3 + 100 \cdot (2.096) = 3 + 209.6 = 212.6$$

Answer: 212.6