Mathematics for Business B

Step-by-Step Solutions

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Hello, I am Jovan Samke!

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My Story

- I am 24-year old guy from Serbia, and I went to **the best math high school in Europe.**
- I did my BSc in Mathematics, had **a 93% GPA**, and stacked up a bunch of awards.
- Right now, I am doing my MSc in Statistics and Data Science at KU Leuven, and **I am the only one who scored a full scholarship**.
- I have done hundreds of tutoring sessions, helping dozens of BBA students succeed, **nearly all of them passed their exams.**

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Chapter 1 - Functions of Two Variables

Chapter 1 - Question 1

Question

Find the domains of the following functions of two variables.

(a) $f(x,y) = \dfrac{xy}{\sqrt{x^2-1}}$

(b) $f(x,y) = \sqrt{(x-1)(2+y)}$

(c) $f(x,y) = \ln\left(4-x^2-y^2\right)$

(d) $f(x,y) = \dfrac{1}{x+y-2}$

(e) $f(x,y) = \sqrt{y-x+1}$

(f) $f(x,y) = \dfrac{1}{\sqrt{xy}}$

(g) $f(x,y) = \log_2\left(x^2+y^2-9\right)$

Solution

(a) Finding the domain of $f(x,y) = \frac{xy}{\sqrt{x^2-1}}$

Tip : Understanding Constraints

- Whenever we have a square root $\sqrt{\text{something}}$, the thing inside must be **zero or positive** (never negative), otherwise, the function is not defined.
- A denominator must **never be zero**, because division by zero is not allowed.

Step 1: Identify constraints

The denominator contains a square root, so we impose the conditions:

$$x^2 - 1 \geq 0$$

$$x^2 - 1 \neq 0$$

Step 2: Solve the first constraint using a sign chart
Rewriting the inequality:

$$x^2 - 1 \geq 0$$

Factoring:

$$(x-1)(x+1) \geq 0$$

Solving using a sign chart:

x		-1		1	
$(x-1)$	$-$	$-$	$-$	0	$+$
$(x+1)$	$-$	0	$+$	$+$	$+$
$(x-1)(x+1)$	$+$	0	$-$	0	$+$

From the sign chart, $(x-1)(x+1) \geq 0$ means:

$$x \leq -1 \quad \text{or} \quad x \geq 1$$

Step 3: Solve the second constraint

$$x^2 - 1 \neq 0$$

This means:

$$(x-1)(x+1) \neq 0$$

So we must exclude $x = \pm 1$.

Step 4: Combine the two conditions

- From Step 2: $x \leq -1$ or $x \geq 1$.
- From Step 3: $x \neq -1$ and $x \neq 1$.

Since both conditions must be true at the same time, we remove $x = \pm 1$, giving:

$$x < -1 \quad \text{or} \quad x > 1$$

Since there are no restrictions on y, it can be any real number.

Final Domain:

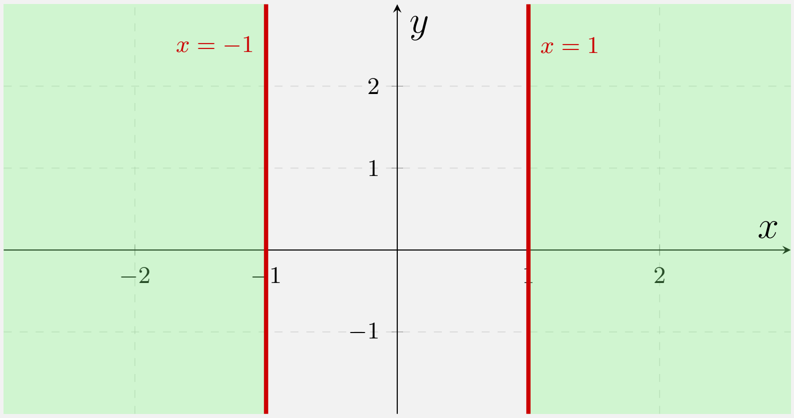
$$\{(x, y) \in \mathbb{R}^2 \mid x < -1 \text{ or } x > 1, \quad y \in \mathbb{R}\}$$

Graphical Representation:

Tip : Understanding Vertical and Horizontal Lines

- The equation $x = a$ represents a **vertical line** at $x = a$ because every point on this line has the same x-value while y can be anything.
- The equation $y = a$ represents a **horizontal line** at $y = a$ because every point on this line has the same y-value while x can be anything.

The valid domain is highlighted in green, while the excluded points ($x = \pm 1$) are marked with red lines.



The vertical red lines at $x = 1$ and $x = -1$ indicate that these values are excluded from the domain.

Answer: The domain is $\{(x, y) \in \mathbb{R}^2 \mid x < -1 \text{ or } x > 1, y \in \mathbb{R}\}$.

(b) Finding the domain of $f(x, y) = \sqrt{(x-1)(2+y)}$

Step 1: Identify constraints
Since the expression under the square root must be non-negative:

$$(x-1)(2+y) \geq 0$$

Step 2: Solve the inequality by analyzing cases
The inequality we need to solve is:

$$(x-1)(2+y) \geq 0$$

Since this is a product of two factors, it is non-negative when:

- Both factors are positive:

$$x - 1 \geq 0 \quad \text{and} \quad 2 + y \geq 0 \quad \Rightarrow \quad x \geq 1 \quad \text{and} \quad y \geq -2.$$

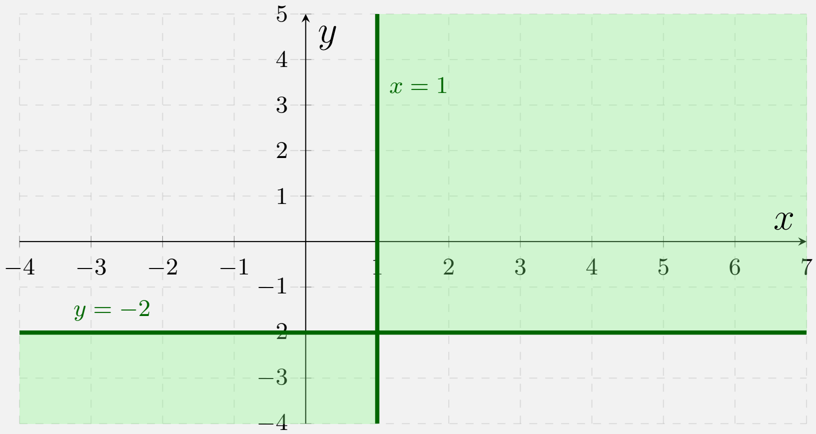
- Both factors are negative:

$$x - 1 \leq 0 \quad \text{and} \quad 2 + y \leq 0 \quad \Rightarrow \quad x \leq 1 \quad \text{and} \quad y \leq -2.$$

Final Domain:

$$\{(x, y) \in \mathbb{R}^2 \mid (x \geq 1, y \geq -2) \text{ or } (x \leq 1, y \leq -2)\}$$

Graphical Representation:
The valid domain is highlighted in green.



Answer: The domain is $\{(x, y) \in \mathbb{R}^2 \mid (x \geq 1, y \geq -2) \text{ or } (x \leq 1, y \leq -2)\}$.

(c) Finding the domain of $f(x, y) = \ln(4 - x^2 - y^2)$

Tip : Understanding Logarithm Constraints

The expression inside a logarithm $\ln(\text{something})$ must be **strictly positive** (greater than zero). It cannot be zero or negative.

Step 1: Identify constraints
Since the logarithm must have a positive argument:

$$4 - x^2 - y^2 > 0$$

Step 2: Solve the inequality
Rearrange:

$$4 > x^2 + y^2$$

Tip : Understanding Circle Equations

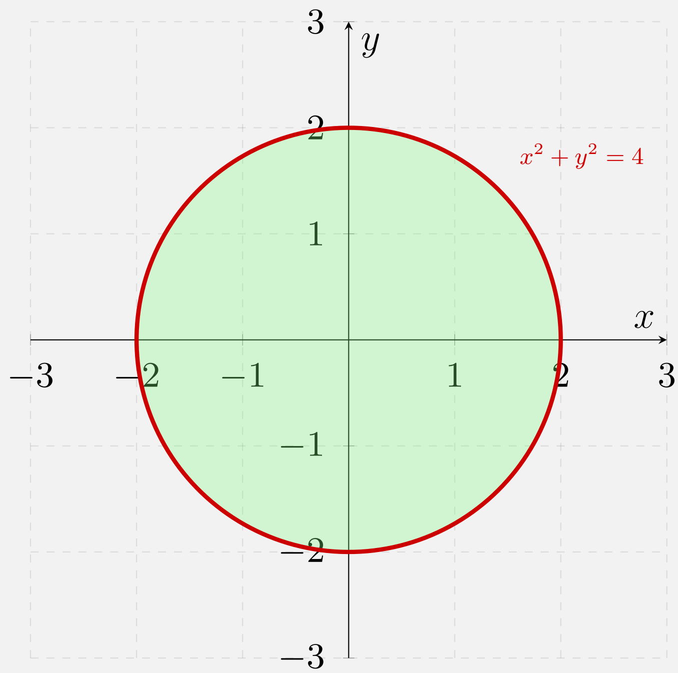
- The equation $x^2 + y^2 = r^2$ represents a **circle** centered at $(0, 0)$ with radius r.
- If the inequality is $x^2 + y^2 < r^2$, it represents the **inside** of the circle.
- If the inequality is $x^2 + y^2 > r^2$, it represents the **outside** of the circle.

Hence, this represents a circle centered at $(0, 0)$ with radius 2, but the inequality is strict ($<$), meaning the points on the circle's edge are **not included** in the domain.

Final Domain:

$$\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 4\}$$

Graphical Representation:
The valid domain is highlighted in green, while the excluded circle boundary is marked in red.



Answer: The domain is $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 4\}$.

(d) Finding the domain of $f(x, y) = \frac{1}{x+y-2}$

Tip : Understanding Denominator Constraints

A denominator must **never be zero**, because division by zero is undefined.

Step 1: Identify constraints
Since the denominator cannot be zero, we impose the condition:

$$x + y - 2 \neq 0$$

Step 2: Solve for the excluded values
Rearrange the equation:

$$y \neq 2 - x$$

Tip : Understanding Linear Equations

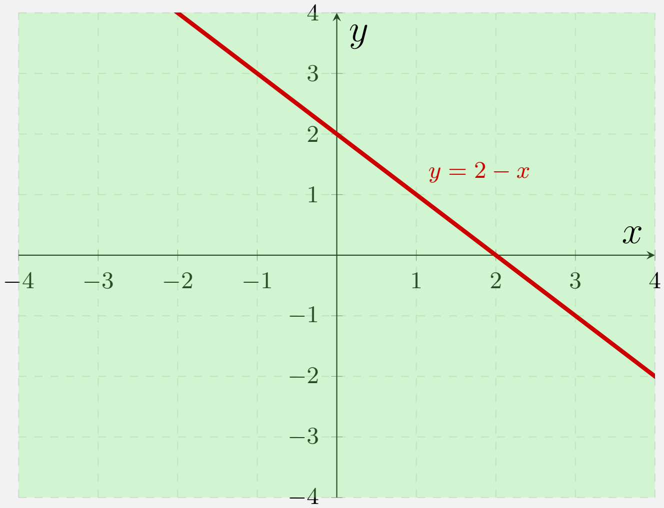
An equation of the form $y = mx + b$ represents a **straight line** with slope m and y-intercept b.

Hence, this equation represents a straight line $y = 2 - x$. Every point on this line is excluded from the domain.

Final Domain:

$$\{(x, y) \in \mathbb{R}^2 \mid y \neq 2 - x\}$$

Graphical Representation:
The valid domain is highlighted in green, while the excluded line $y = 2 - x$ is marked in red.



Answer: The domain is $\{(x, y) \in \mathbb{R}^2 \mid y \neq 2 - x\}$.

(e) Finding the domain of $f(x, y) = \sqrt{y - x + 1}$

Step 1: Identify constraints
Since the square root must have a non-negative argument:

$$y - x + 1 \geq 0$$

Step 2: Solve for the valid region
Rearrange the inequality:

$$y \geq x - 1$$

Tip : Understanding Linear Equations

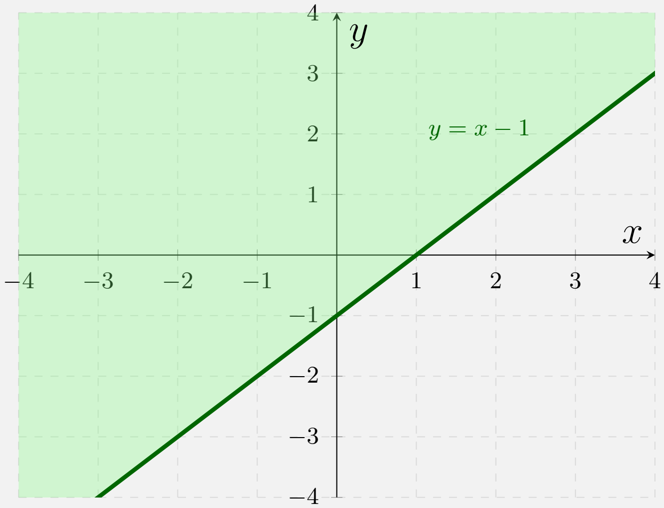
- An equation of the form $y = mx + b$ represents a **straight line** with slope m and y-intercept b.
- When the inequality is $y \geq mx + b$, the valid region is **above or on the line**.
- When the inequality is $y \leq mx + b$, the valid region is **below or on the line**.

Hence, this represents the region **above or on** the line $y = x - 1$, since the inequality includes $\geq$, meaning points on the line itself are included in the domain.

Final Domain:

$$\{(x, y) \in \mathbb{R}^2 \mid y \geq x - 1\}$$

Graphical Representation:
The valid domain is highlighted in green.



Answer: The domain is $\{(x, y) \in \mathbb{R}^2 \mid y \geq x - 1\}$.

(f) Finding the domain of $f(x, y) = \frac{1}{\sqrt{xy}}$

Step 1: Identify constraints
Since we have both a square root and a denominator, we must check two conditions separately:

- The expression inside the square root must be non-negative:

$$xy \geq 0$$

- The denominator cannot be zero:

$$xy \neq 0$$

Step 2: Solve for the valid regions
The condition $xy \geq 0$ means x and y must have the same sign:

- If $x > 0$, then $y \geq 0$ (both positive or zero).
- If $x < 0$, then $y \leq 0$ (both negative or zero).

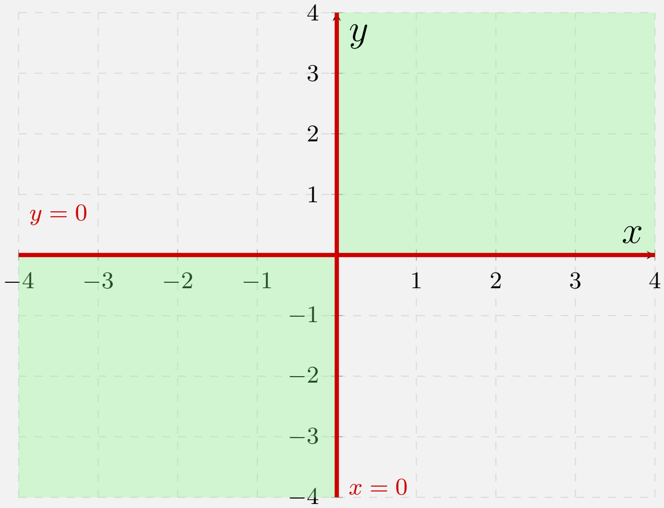
However, the additional condition $xy \neq 0$ means that we must exclude the x- and y-axes ($x = 0$ and $y = 0$), since at least one variable being zero would make $xy = 0$.

Final Domain:

$$\{(x,y) \in \mathbb{R}^2 \mid xy > 0\}$$

This means the function is defined in the first and third quadrants, but the x- and y-axes are excluded.

Graphical Representation: The valid domain is highlighted in green, while the excluded x- and y-axes are marked in red.



Answer: The domain is $\{(x,y) \in \mathbb{R}^2 \mid xy > 0\}$

(g) Finding the domain of $f(x,y) = \log_2(x^2 + y^2 - 9)$

Step 1: Identify constraints
Since the logarithm must have a positive argument:

$$x^2 + y^2 - 9 > 0$$

Step 2: Solve the inequality
Rearrange:

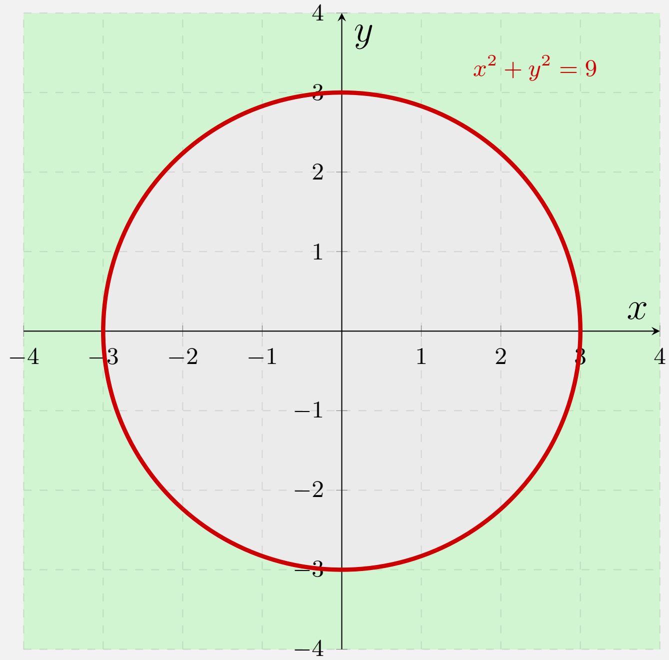
$$x^2 + y^2 > 9$$

This represents the outside of a circle centered at $(0,0)$ with radius 3. Since the inequality is strict ($>$), the points on the circle itself are not included in the domain.

Final Domain:

$$\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 > 9\}$$

Graphical Representation:
The valid domain is highlighted in green, while the excluded circle boundary is marked in red.



Answer: The domain is $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 > 9\}$.

Chapter 1 - Question 2

Question

Make the sign chart for the following continuous functions of two variables.

(a) $f(x, y) = 2x - 3y + 6$

(b) $f(x, y) = (x + 1)^2 + y^2 - 1$

(c) $f(x, y) = x^2 - xy + x$

(d) $f(x, y) = x^2 + y^2 - x$

(e) $f(x, y) = \dfrac{\sqrt{x - y}}{y}$

(f) $f(x, y) = \dfrac{\ln(x - y)}{y}$

Solution

(a) Finding the sign chart of $f(x, y) = 2x - 3y + 6$

Tip : Key Steps for Sign Charts

To determine where a function of two variables is positive or negative, follow these three steps:

1. **Find the domain** – Check for any restrictions on x and y.
2. **Find where the function is zero** – Solve $f(x, y) = 0$ to identify zeroes. These are going to serve as boundary lines or curves.
3. **Find the sign in each region** – Pick a test point in each region to determine whether the function is positive or negative.

Step 1: Find the domain

The given function is a linear equation, which is continuous and defined for all $(x, y) \in \mathbb{R}^2$, so the domain is:

$$\mathbb{R}^2$$

Step 2: Find where the function is zero

Setting $f(x, y) = 0$:

$$2x - 3y + 6 = 0$$

Rearrange for y:

$$y = \frac{2}{3}x + 2$$

This is a straight line that separates the plane into two regions.

Tip : Understanding Sign Charts for Linear Functions

For a function of the form $f(x, y) = ax + by + c$:

- The function is **zero** on the line $ax + by + c = 0$.
- The function is **positive** on one side of the line and **negative** on the other.
- To determine which side is positive or negative, pick a test point (e.g., $(0, 0)$) and check the sign of $f(x, y)$.

Step 3: Identify the sign in each region

We select a test point to determine the sign of $f(x, y)$ in different regions.

- Test Point 1: $(0, 0)$

$$f(0, 0) = 2 \cdot (0) - 3 \cdot (0) + 6 = 6 > 0$$

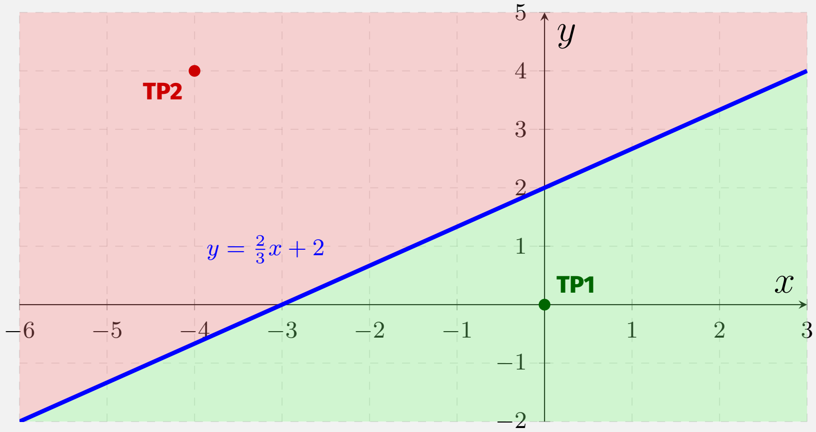
  Hence, the region containing $(0, 0)$ is positive (green area).

- Test Point 2: $(-4, 4)$ (a point in the opposite region)

$$f(-4, 4) = 2 \cdot (-4) - 3 \cdot (4) + 6 = -14 < 0$$

  Hence, the opposite region is negative (red area).

Answer: The green region represents where the function is positive, the red region where it is negative, and the blue line where it is zero.



(b) Finding the sign chart of $f(x, y) = (x+1)^2 + y^2 - 1$

Step 1: Find the domain

The given function is a quadratic expression in x and y, and it is defined for all $(x, y) \in \mathbb{R}^2$, so the domain is:

$$\mathbb{R}^2$$

Step 2: Find where the function is zero

Setting $f(x, y) = 0$:

$$(x+1)^2 + y^2 - 1 = 0$$

Rearrange:

$$(x+1)^2 + y^2 = 1$$

This represents a circle centered at $(-1, 0)$ with radius 1.

Tip : Understanding Sign Charts for Quadratic Functions

For a function of the form $f(x, y) = (\text{expression})^2 + (\text{expression})^2 - c$:

- The function is **zero** on the boundary (circle) equation.
- The function is **positive** outside the boundary and **negative** inside.
- To verify, pick test points inside and outside the boundary.

Step 3: Identify the sign in each region

We select a test point to determine the sign of $f(x, y)$ in different regions.

- Test Point 1: $(-1, 0)$ (center of the circle)

  $$f(-1, 0) = (-1+1)^2 + 0^2 - 1 = -1 < 0$$

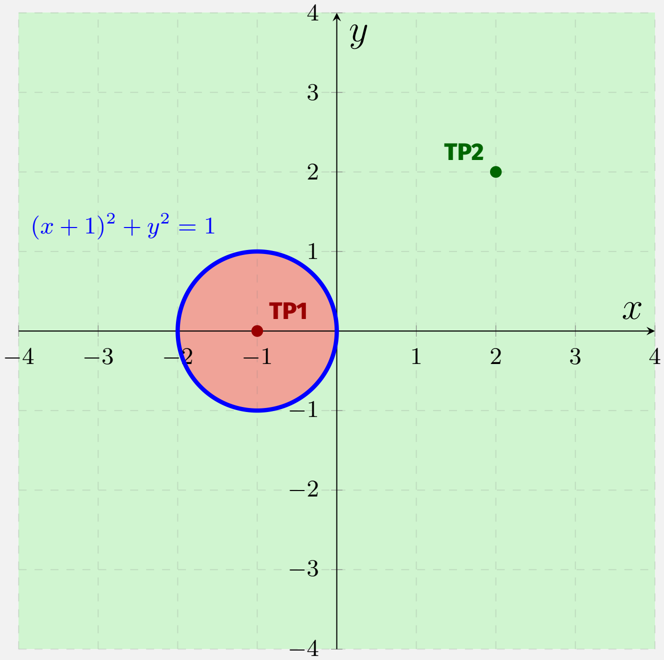
  Hence, the region inside the circle is negative (red area).

- Test Point 2: $(2, 2)$ (a point outside the circle)

  $$f(2, 2) = (2+1)^2 + 2^2 - 1 = 12$$

  Hence, the region outside the circle is positive (green area).

Answer: The green region represents where the function is positive, the red region where it is negative, and the blue circle where it is zero.



(c) Finding the sign chart of $f(x, y) = x^2 - xy + x$

Step 1: Find the domain
The given function is a quadratic polynomial in x and y, which is defined for all $(x, y) \in \mathbb{R}^2$, so the domain is:

$$\mathbb{R}^2$$

Step 2: Find where the function is zero
Setting $f(x, y) = 0$:

$$x^2 - xy + x = 0$$

Factorizing:

$$x(x - y + 1) = 0$$

This gives two boundary lines:

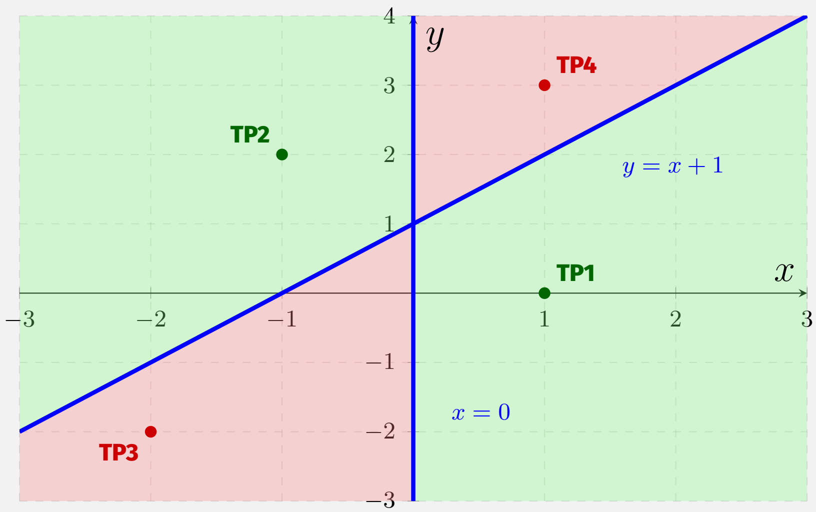
$$x = 0 \quad \text{or} \quad y = x + 1$$

These lines separate the plane into different regions.

Step 3: Identify the sign in each region
We select test points to determine the sign of $f(x, y)$ in different regions.

- Test Point 1: $(1, 0)$
$$f(1, 0) = 1 - 0 + 1 = 2 > 0$$
Hence, the region containing $(1, 0)$ is positive (green area).
- Test Point 2: $(-1, 2)$
$$f(-1, 2) = 1 + 2 - 1 = 2 > 0$$
Hence, this region is also positive (green area).
- Test Point 3: $(-2, -2)$
$$f(-2, -2) = 4 - 4 - 2 = -2 < 0$$
Hence, this region is negative (red area).
- Test Point 4: $(1, 3)$
$$f(1, 3) = 1 - 3 + 1 = -1 < 0$$
Hence, this region is also negative (red area).

Answer: The green region represents where the function is positive, the red region where it is negative, and the blue lines where it is zero.



(d) Finding the sign chart of $f(x, y) = x^2 + y^2 - x$

Step 1: Find the domain
The function is a polynomial in x and y, which means it is defined for all $(x, y) \in \mathbb{R}^2$, so the domain is:
$$\mathbb{R}^2$$

Step 2: Find where the function is zero
Setting $f(x, y) = 0$:
$$x^2 + y^2 - x = 0$$

Tip : Rewriting Equations into Standard Circle Form

An equation of the form:
$$x^2 + y^2 + mx + ny + l = 0$$
can be rewritten into the standard circle equation:
$$(x - a)^2 + (y - b)^2 = r^2$$
where (a, b) is the center and r is the radius. To do this:

1. Group the x- and y-terms separately.
2. Complete the square on the variable that has a linear term.
3. Rewrite in standard circle form.

This helps us identify the center and radius of the circle.

We want to rewrite this equation in the standard form of a circle:
$$(x - a)^2 + (y - b)^2 = r^2$$
Since there is no linear y-term, we only need to complete the square for x. We rewrite:
$$x^2 - x + y^2 = 0$$
To complete the square for $x^2 - x$, we take half the coefficient of x, square it, and add/subtract:
$$x^2 - x = x^2 - 2 \cdot \frac{1}{2} \cdot x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = (x - \frac{1}{2})^2 - \frac{1}{4}$$
Substituting this into the equation:
$$(x - \frac{1}{2})^2 - \frac{1}{4} + y^2 = 0 \quad \Rightarrow \quad (x - \frac{1}{2})^2 + y^2 = \frac{1}{4}$$
Thus, the function is zero along a circle centered at $(\frac{1}{2}, 0)$ with radius $\frac{1}{2}$.

Step 3: Identify the sign in each region
We select test points to determine the sign of $f(x, y)$ in different regions.

- Test Point 1: $\left(\frac{1}{2}, 0\right)$ (center of the circle)

$$f\left(\frac{1}{2}, 0\right) = \left(\frac{1}{2}\right)^2 + 0^2 - \frac{1}{2} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} < 0$$

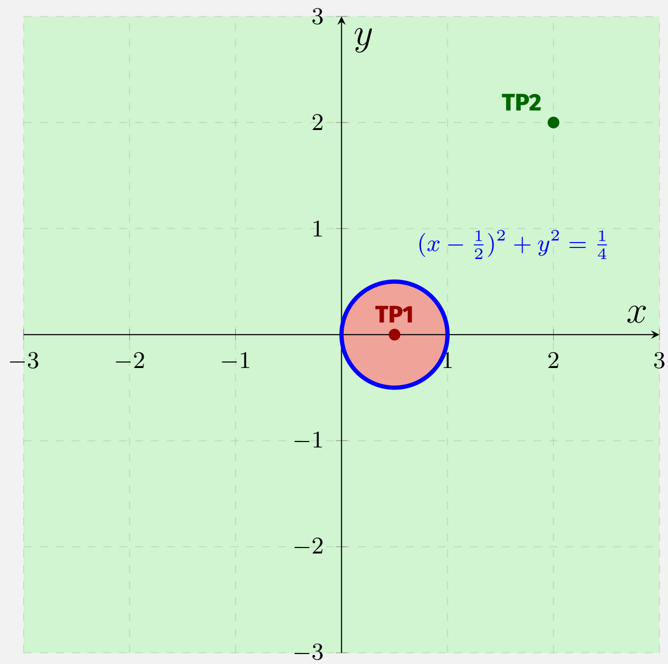
  Hence, the region inside the circle is negative (red area).

- Test Point 2: $(2, 2)$ (outside the circle)

$$f(2, 2) = 2^2 + 2^2 - 2 = 4 + 4 - 2 = 6 > 0$$

  Hence, the region outside the circle is positive (green area).

Answer: The green region represents where the function is positive, the red region where it is negative, and the blue circle where it is zero.



(e) Finding the sign chart of $f(x, y) = \frac{\sqrt{x-y}}{y}$

Step 1: Find the domain

Since the function involves both a square root and a denominator, we must analyze both constraints separately.

Tip : Understanding Square Root and Denominator Constraints

For a function of the form $f(x, y) = \frac{\sqrt{\text{something}}}{\text{denominator}}$:

- The expression inside the square root must be non-negative.
- The denominator cannot be zero.

Both conditions must hold at the same time.

- The square root constraint:

$$x - y \geq 0 \quad \Rightarrow \quad x \geq y$$

- The denominator constraint:

$$y \neq 0$$

Since both conditions must be satisfied simultaneously, we conclude:

$$\text{Domain: } \{(x, y) \in \mathbb{R}^2 \mid x \geq y, \quad y \neq 0\}$$

Step 2: Find where the function is zero
The function is zero when:

$$\sqrt{x - y} = 0 \quad \Rightarrow \quad x - y = 0 \quad \Rightarrow \quad x = y$$

This means the function is zero along the line $x = y$.

Step 3: Identify the sign in each region
We test points from two valid regions to determine the sign of $f(x, y)$.

- Test Point 1: $(2, 1)$ (above the zero line)

$$f(2, 1) = \frac{\sqrt{2 - 1}}{1} = 1 > 0$$

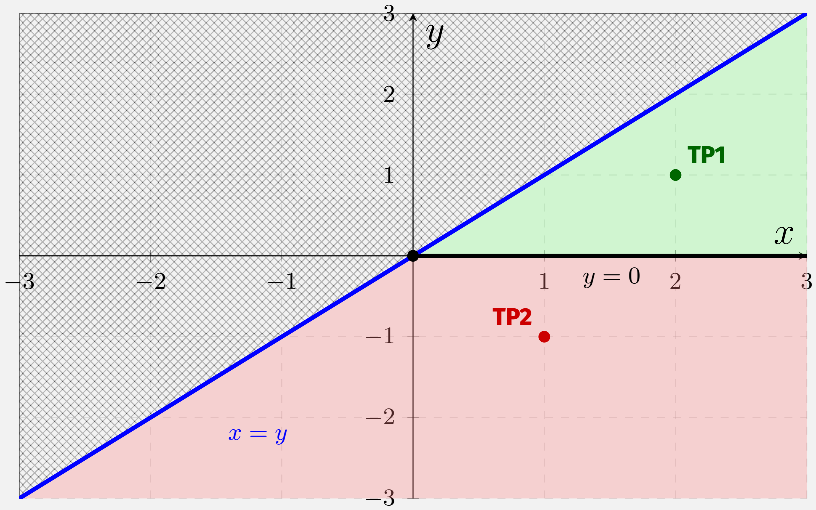
  Hence, this region is positive (green area).

- Test Point 2: $(1, -1)$ (below the zero line)

$$f(1, -1) = \frac{\sqrt{1 - (-1)}}{-1} = \frac{\sqrt{2}}{-1} < 0$$

  Hence, this region is negative (red area).

Answer: The green region represents where the function is positive, the red region where it is negative, the blue line where it is zero, and the black-shaded region (including the x-axis) shows where the function is not defined.



(f) Finding the sign chart of $f(x, y) = \frac{\ln(x-y)}{y}$

Step 1: Find the domain

Since the function involves a logarithm and a denominator, we must analyze both constraints separately.

Tip : Understanding Logarithm and Denominator Constraints

For a function of the form $f(x, y) = \frac{\ln(\text{something})}{\text{denominator}}$:

- The argument of the logarithm must be **strictly positive**.
- The denominator cannot be zero.

Both conditions must be satisfied at the same time.

- The logarithm constraint:

$$x - y > 0 \quad \Rightarrow \quad x > y$$

- The denominator constraint:

$$y \neq 0$$

Since both conditions must hold simultaneously, we conclude:

$$\text{Domain: } \{(x, y) \in \mathbb{R}^2 \mid x > y, \quad y \neq 0\}$$

Step 2: Find where the function is zero
The function is zero when:

$$\ln(x - y) = 0 \quad \Rightarrow \quad x - y = 1 \quad \Rightarrow \quad y = x - 1$$

Thus, the function is zero along the line $y = x - 1$.

Step 3: Identify the sign in each region
We select test points from the four valid regions to determine the function's sign.

- Test Point 1: $(-1, -1.5)$ (bottom-left region)

$$f(-1, -1.5) = \frac{\ln(-1 - (-1.5))}{-1.5} = \frac{\ln 0.5}{-1.5} > 0$$

  Hence, this region is positive (green area).

- Test Point 2: $(1, -1)$ (bottom-right region)

$$f(1, -1) = \frac{\ln(1 - (-1))}{-1} = \frac{\ln 2}{-1} < 0$$

  Hence, this region is negative (red area).

- Test Point 3: $(3, 1)$ (top-right region)

$$f(3, 1) = \frac{\ln(3 - 1)}{1} = \ln 2 > 0$$

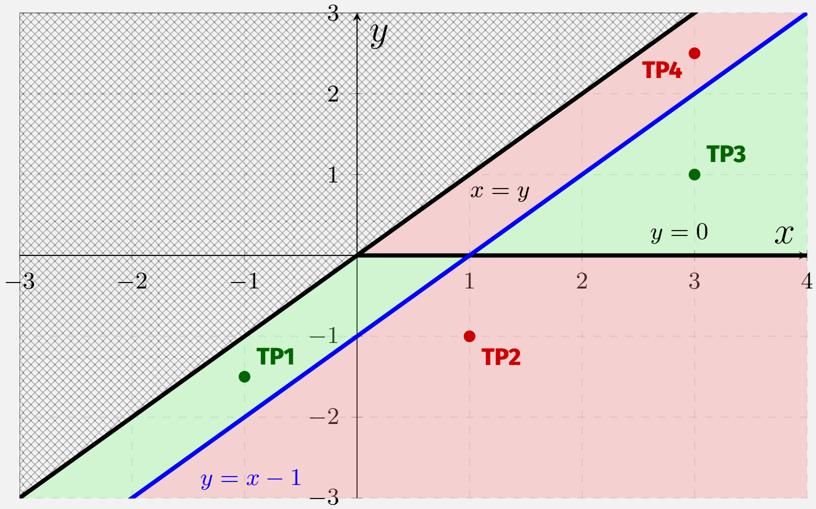
  Hence, this region is positive (green area).

- Test Point 4: $(3, 2.5)$ (topmost region)

$$f(3, 2.5) = \frac{\ln(3 - 2.5)}{2.5} = \frac{\ln 0.5}{2.5} < 0$$

  Hence, this region is negative (red area).

Answer: The green regions represent where the function is positive, the red regions where it is negative, the blue line where it is zero, and the black-shaded region (including the x-axis) shows where the function is not defined.



Chapter 1 - Question 3

Question

Solve the following inequalities.

(a) $2x + 6 \leq 3y$

(b) $y > y \cdot \log_2 x$

Solution

(a) Solving the inequality $2x + 6 \leq 3y$

Tip : Solving Inequalities in Two Variables

To solve an inequality of the form $f(x, y) \leq 0$, follow these steps:

1. **Find the domain**: Identify any restrictions on x and y
2. **Rewrite the inequality**: Move all terms to one side, to get $f(x, y) \leq 0$.
3. **Find zeroes**: Solve $f(x, y) = 0$ to determine the dividing line or curve.
4. **Identify the solution region**: Pick a test point in each region and evaluate $f(x, y)$ to determine where the inequality holds.

Step 1: Find the domain

The given function is a linear equation, which is continuous and defined for all $(x, y) \in \mathbb{R}^2$. Hence, the domain is:

$$\mathbb{R}^2$$

Step 2: Rewrite the inequality

We express the inequality in the form $f(x, y) \leq 0$:

$$2x + 6 - 3y \leq 0$$

This means we analyze the sign of the function:

$$f(x, y) = 2x + 6 - 3y$$

Step 3: Find zeroes

Setting $f(x, y) = 0$:

$$2x + 6 - 3y = 0$$

Solving for y:

$$y = \frac{2}{3}x + 2$$

This is a straight line that separates the plane into two regions. To determine which side of the line satisfies the inequality, pick a test point and check its sign.

Step 4: Identify the solution region

We select a test point to determine the sign of $f(x, y)$ in different regions.

- Test Point: $(0, 0)$

  $$f(0, 0) = 2(0) + 6 - 3(0) = 6 > 0$$

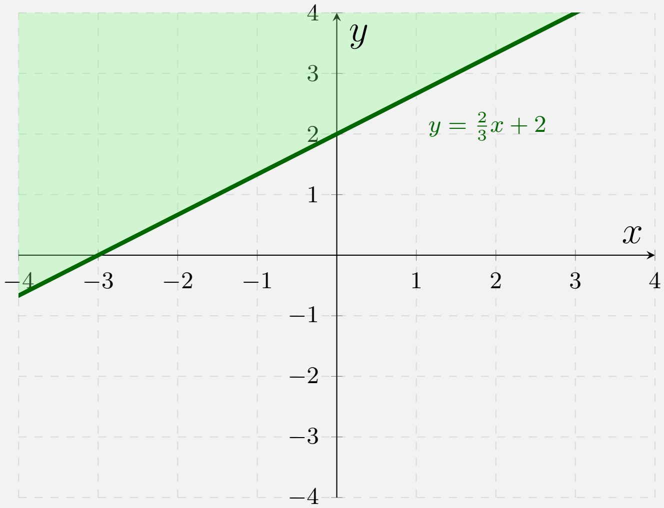
  Since $f(0, 0) > 0$, the region containing $(0, 0)$ is not part of the solution.

- Test Point: $(-3, 3)$

  $$f(-3, 3) = 2(-3) + 6 - 3(3) = -6 + 6 - 9 = -9 < 0$$

  Since $f(-3, 3) < 0$, this region satisfies the inequality.

Answer: The green region represents where the inequality $f(x, y) \leq 0$ holds, and the dark green line represents the boundary where $f(x, y) = 0$.



(b) Solving the inequality $y > y \log_2 x$

Step 1: Find the domain

Since the function involves $\log_2 x$, we must ensure that $x > 0$ because the logarithm is only defined for positive values of x. Thus, the domain is:

$$\{(x, y) \in \mathbb{R}^2 \mid x > 0\}$$

Step 2: Rewrite the inequality

Rewriting the given inequality:

$$y > y \log_2 x$$

Factor out y:

$$y(1 - \log_2 x) > 0$$

This inequality is satisfied when:

- $y > 0$ and $1 - \log_2 x > 0$ (i.e., $x < 2$)
- $y < 0$ and $1 - \log_2 x < 0$ (i.e., $x > 2$)

Step 3: Find zeroes

Setting $f(x, y) = 0$:

$$y \cdot (1 - \log_2 x) = 0$$

This occurs when either:

$$y = 0 \quad \text{or} \quad x = 2$$

Thus, the boundary consists of the x-axis and the vertical line $x = 2$.

Step 4: Identify the solution region
We select test points to determine the sign of $f(x, y)$ in different regions.

- Test Point 1: $(1, 2)$ (left of $x = 2$, above $y = 0$)

$$f(1, 2) = 2(1 - \log_2 1) = 2(1 - 0) = 2 > 0$$

  Hence, this region is positive (green area).

- Test Point 2: $(3, 2)$ (right of $x = 2$, above $y = 0$)

$$f(3, 2) = 2(1 - \log_2 3) = 2(1 - \log_2 3) < 0$$

  Since $\log_2 3 > 1$, this region is negative (red boundary).

- Test Point 3: $(1, -2)$ (left of $x = 2$, below $y = 0$)

$$f(1, -2) = (-2)(1 - \log_2 1) = -2(1 - 0) = -2 < 0$$

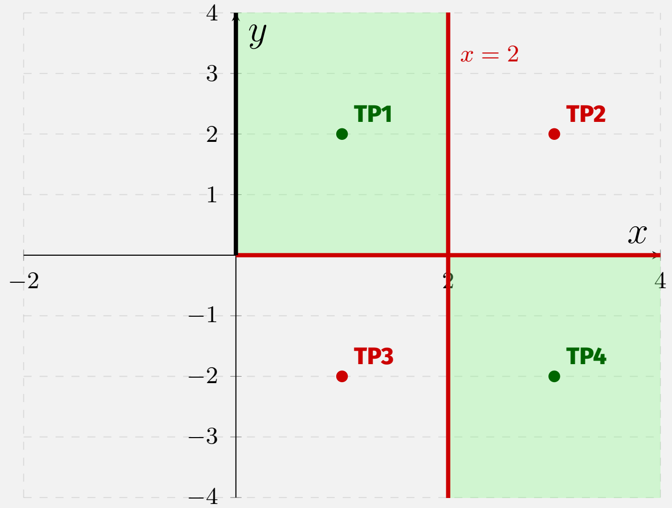
  Hence, this region is negative (red boundary).

- Test Point 4: $(3, -2)$ (right of $x = 2$, below $y = 0$)

$$f(3, -2) = (-2)(1 - \log_2 3) = -2(1 - \log_2 3) > 0$$

  Since $\log_2 3 > 1$, this region is positive (green area).

Answer: The green region represents where the function is positive, red lines indicate where the function is zero, and black lines indicate where the function is undefined .



Chapter 1 - Question 4

Question

Graph the contour lines of levels $-1, -0.5, 0, 0.5,$ and 1 of the function

$$f(x,y) = \frac{1}{x+y-2}$$

on the (x,y)-plane.

Solution

Tip : Steps for Finding Contour Lines

To find contour lines of a function $f(x,y)$, follow these steps:

1. **Find the domain** – Determine where $f(x,y)$ is defined.
2. **Set up the contour equation** – Solve $f(x,y) = a$ for a general constant a.
3. **Check when it exists** – Identify which values of a lead to valid contour lines.
4. **Draw the contour lines** – Plot the contour lines for the given levels.

Step 1: Find the domain
The function $f(x,y) = \frac{1}{x+y-2}$ is undefined where the denominator is zero:

$$x + y - 2 = 0 \quad \Rightarrow \quad y = 2 - x$$

Thus, the domain is:

$$\{(x,y) \in \mathbb{R}^2 \mid x + y \neq 2\}$$

Step 2: Set up the contour equation
Contour lines correspond to solving $f(x,y) = a$ for given values of a:

$$\frac{1}{x+y-2} = a \quad \Rightarrow \quad a(x+y-2) = 1$$

Step 3: Check for existence
If $a = 0$, the equation is not defined, so no contour line exists. If $a \neq 0$, we can divide by a and solve for y:

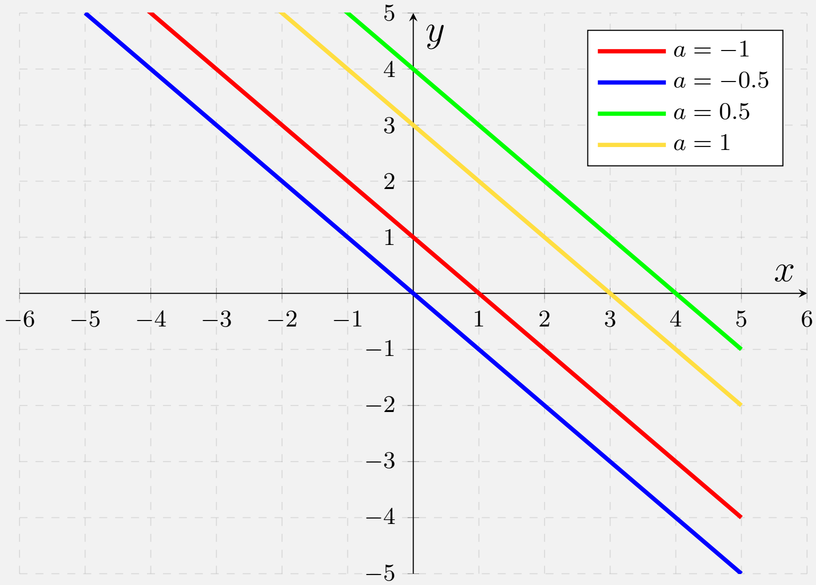
$$y = -x + 2 + \frac{1}{a}$$

Step 4: Plot the contour lines

We now plot the contour lines for different values of a:

$$\begin{aligned} a = -1 \quad &\Rightarrow \quad y = -x + 2 - 1 \quad \Rightarrow \quad y = -x + 1 \\ a = -0.5 \quad &\Rightarrow \quad y = -x + 2 - 2 \quad \Rightarrow \quad y = -x \\ a = 0.5 \quad &\Rightarrow \quad y = -x + 2 + 2 \quad \Rightarrow \quad y = -x + 4 \\ a = 1 \quad &\Rightarrow \quad y = -x + 2 + 1 \quad \Rightarrow \quad y = -x + 3 \end{aligned}$$

Since $a = 0$ leads to an undefined equation, no contour line exists for this case.



Answer: The contour lines are given by $y = -x + 2 + \frac{1}{a}$, where $a \neq 0$. There is no contour line for $a = 0$ since the function is undefined in that case.

Chapter 1 - Question 5

Question

Graph the contour lines of levels $-2, -1, 0, 1, 2$ of the function

$$f(x, y) = \sqrt{y - x + 1}$$

on the (x, y)-plane.

Solution

Step 1: Find the domain
The function $f(x, y) = \sqrt{y - x + 1}$ involves a square root, which requires the argument to be non-negative:

$$y - x + 1 \geq 0 \quad \Rightarrow \quad y \geq x - 1$$

Thus, the domain is:

$$\{(x, y) \in \mathbb{R}^2 \mid y \geq x - 1\}$$

Step 2: Set up the contour equation
Contour lines correspond to solving $f(x, y) = a$ for given values of a:

$$\sqrt{y - x + 1} = a \quad \Rightarrow \quad a^2 = y - x + 1$$

Step 3: Check for existence
If $a < 0$, the equation does not exist since the square root function is non-negative.
If $a \geq 0$, we can solve for y:

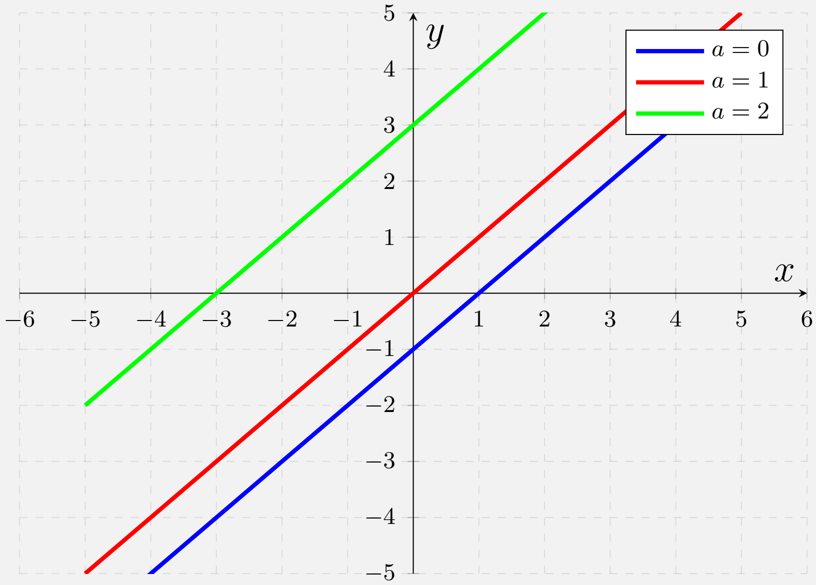
$$y = x - 1 + a^2$$

Step 4: Plot the contour lines

We now plot the contour lines for different values of a:

$$\begin{aligned} a = 0 \quad &\Rightarrow \quad y = x - 1 \\ a = 1 \quad &\Rightarrow \quad y = x - 1 + 1 \quad \Rightarrow \quad y = x \\ a = 2 \quad &\Rightarrow \quad y = x - 1 + 4 \quad \Rightarrow \quad y = x + 3 \end{aligned}$$

Since $a < 0$ is not possible, no contour lines exist for those cases.



Answer: The contour lines are given by $y = x - 1 + a^2$, where $a \geq 0$. There are no contour lines for $a < 0$ since the function is not defined for those values.

Chapter 1 - Question 6

Question

Graph the contour line of level 4 of the function

$$f(x, y) = \log_2(x^2 + y^2 - 9)$$

on the (x, y)-plane.

Solution

Step 1: Find the domain

The function $f(x, y) = \log_2(x^2 + y^2 - 9)$ is defined when its argument is positive:

$$x^2 + y^2 - 9 > 0 \quad \Rightarrow \quad x^2 + y^2 > 9$$

Thus, the domain consists of all points outside the circle $x^2 + y^2 = 9$.

Step 2: Set up the contour equation

Contour lines correspond to solving $f(x, y) = a$ for given values of a:

$$\log_2(x^2 + y^2 - 9) = a$$

Rearranging:

$$x^2 + y^2 - 9 = 2^a$$

Step 3: Check for existence
For the equation to be valid, the right-hand side must be positive, meaning:

$$2^a > 0$$

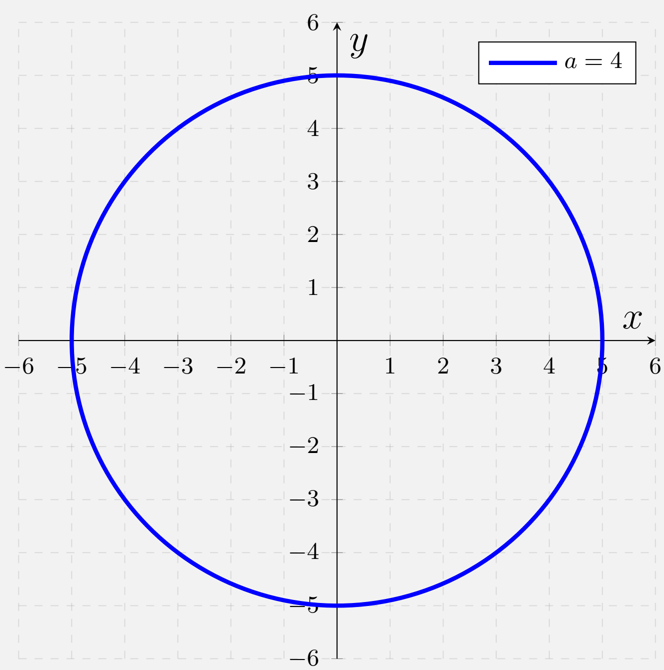
Since 2^a is always positive for all real a, contour lines exist for all values of a.

Step 4: Plot the contour lines

We now plot the contour line for $a = 4$:

$$2^4 = x^2 + y^2 - 9 \quad \Rightarrow \quad x^2 + y^2 = 25$$

which represents a circle centered at $(0, 0)$ with radius 5.



Answer: The contour line for $a = 4$ is given by $x^2 + y^2 = 25$, which is a circle of radius 5 centered at $(0, 0)$. Contour lines exist for all real a.